

NCERT Solutions for Class 10 Maths Unit 6

Triangles Class 10

Unit 6 Triangles Exercise 6.1, 6.2, 6.3, 6.4, 6.5, 6.6 Solutions

Exercise 6.1 : Solutions of Questions on Page Number : 122

Q1 :

Fill in the blanks using correct word given in the brackets:-

- (i) All circles are _____. (congruent, similar)
- (ii) All squares are _____. (similar, congruent)
- (iii) All _____ triangles are similar. (isosceles, equilateral)
- (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____ and (b) their corresponding sides are _____. (equal, proportional)

Answer :

- (i) Similar
- (ii) Similar
- (iii) Equilateral
- (iv) (a) Equal
- (b) Proportional

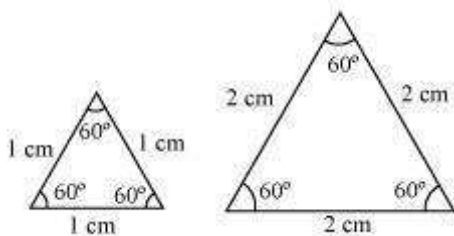
Q2 :

Give two different examples of pair of

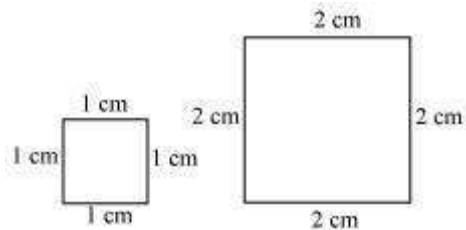
- (i) Similar figures
- (ii) Non-similar figures

Answer :

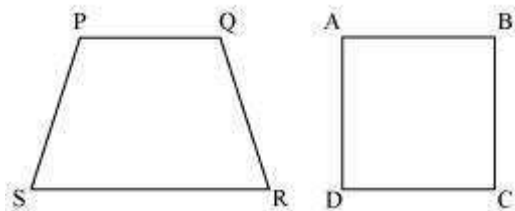
- (i) Two equilateral triangles with sides 1 cm and 2 cm



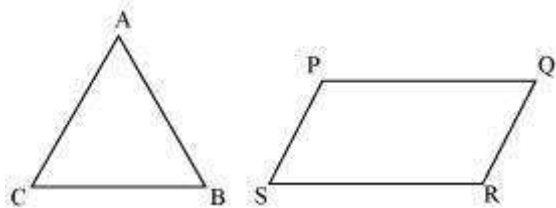
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

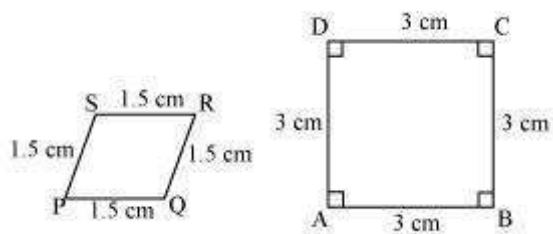


Triangle and parallelogram



Q3 :

State whether the following quadrilaterals are similar or not:



Answer :

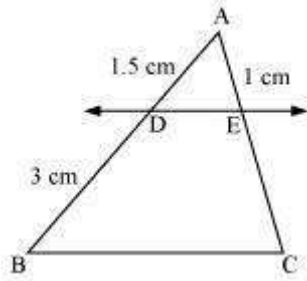
Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Exercise 6.2 : Solutions of Questions on Page Number : 128

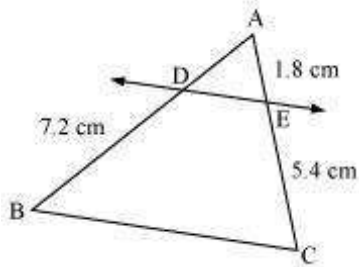
Q1 :

In figure.6.17. (i) and (ii), $DE \parallel BC$. Find EC in (i) and AD in (ii).

(i)

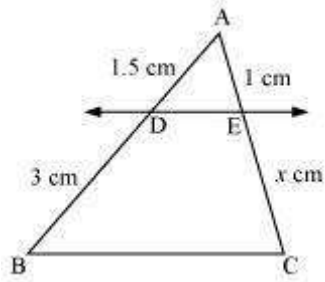


(ii)



Answer :

(i)



Let $EC = x$ cm

It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

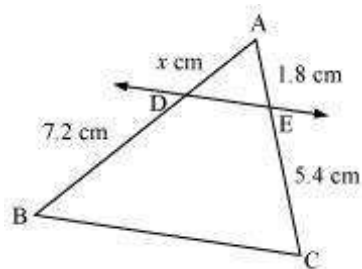
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore EC = 2 \text{ cm}$$

(ii)



Let $AD = x$ cm

It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$$\therefore AD = 2.4 \text{ cm}$$

Q2 :

E and F are points on the sides PQ and PR respectively of a ΔPQR . For each of the following cases, state whether $EF \parallel QR$.

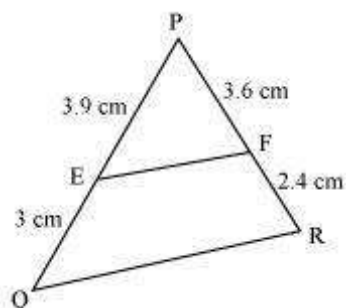
(i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm

(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm

(iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.63$ cm

Answer :

(i)



Given that, $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm, $FR = 2.4$ cm

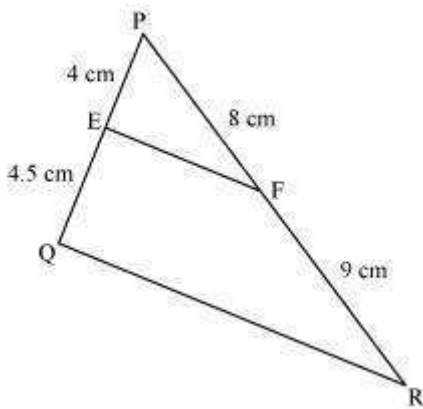
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

$$\text{Hence, } \frac{PE}{EQ} \neq \frac{PF}{FR}$$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

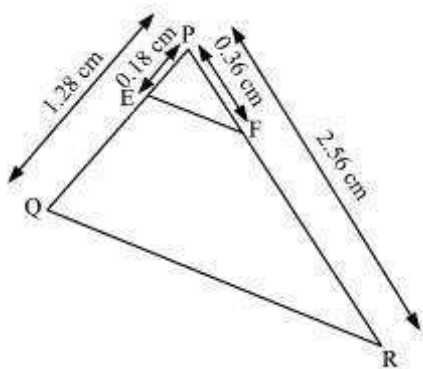
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

$$\text{Hence, } \frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

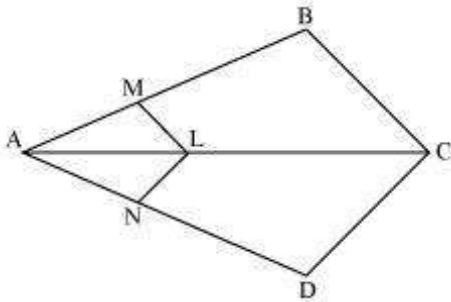
$$\text{Hence, } \frac{PE}{PQ} = \frac{PF}{PR}$$

Therefore, EF is parallel to QR.

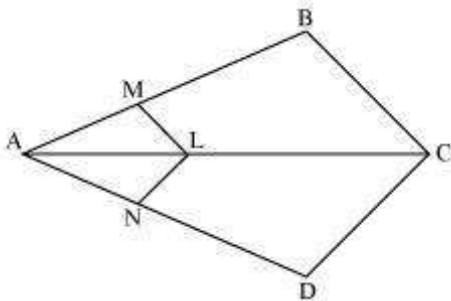
Q3 :

In the following figure, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}.$$



Answer :



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \quad (i)$$

Similarly, $LN \parallel CD$

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \quad (ii)$$

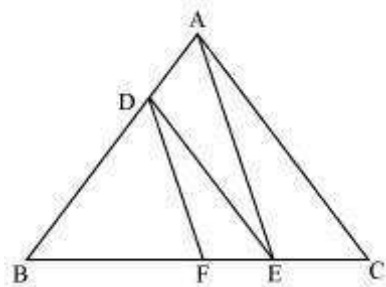
From (i) and (ii), we obtain

$$\frac{AM}{AB} = \frac{AN}{AD}$$

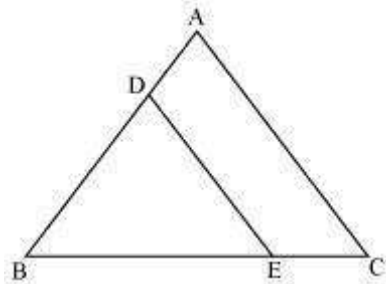
Q4 :

In the following figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

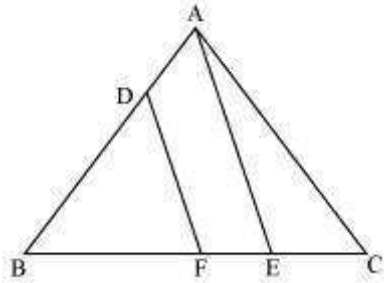


Answer :



In $\triangle ABC$, $DE \parallel AC$

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \quad (\text{Basic Proportionality Theorem}) \quad (i)$$



In $\triangle BAE$, $DF \parallel AE$

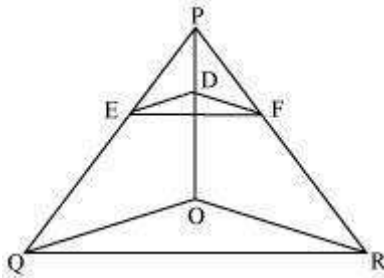
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \quad (\text{Basic Proportionality Theorem}) \quad (ii)$$

From (i) and (ii), we obtain

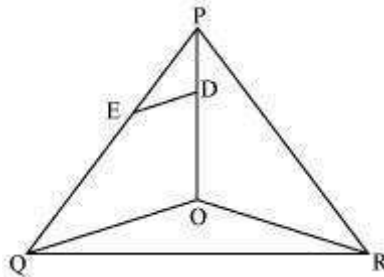
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Q5 :

In the following figure, $DE \parallel OQ$ and $DF \parallel OR$, show that $EF \parallel QR$.

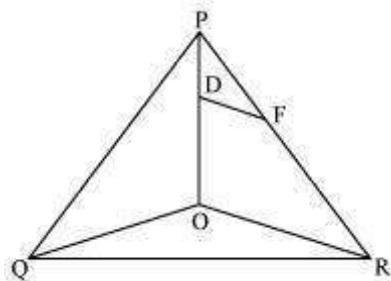


Answer :



In $\triangle POQ$, $DE \parallel OQ$

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (i)$$



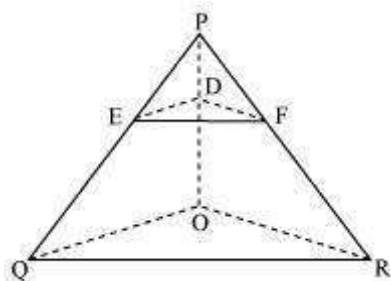
In $\triangle POR$, $DF \parallel OR$

$$\therefore \frac{PF}{FR} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

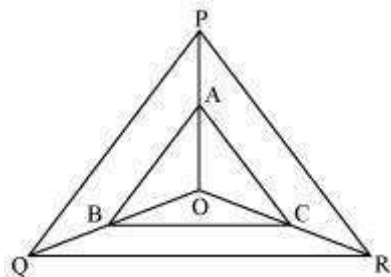
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore EF \parallel QR$ (Converse of basic proportionality theorem)

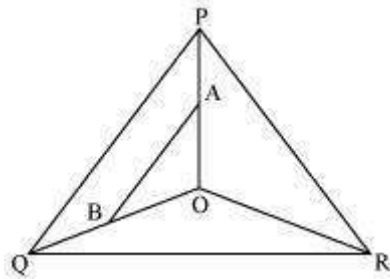


Q6 :

In the following figure, A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.

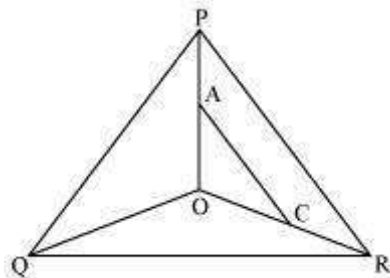


Answer :



In ΔPOQ , $AB \parallel PQ$

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \quad (\text{Basic proportionality theorem}) \quad (i)$$



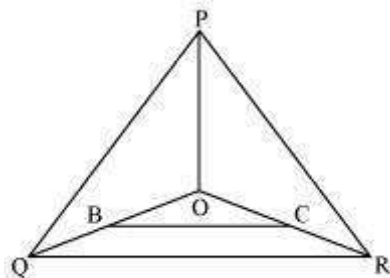
In ΔPOR , $AC \parallel PR$

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \quad (\text{By basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

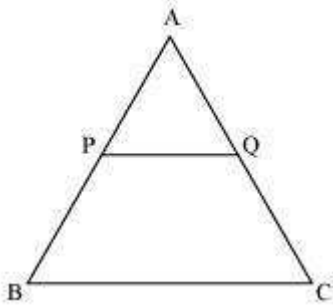
$$\therefore BC \parallel QR \quad (\text{By the converse of basic proportionality theorem})$$



Q7 :

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer :



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \quad (\text{P is the mid-point of AB. } \therefore AP = PB)$$

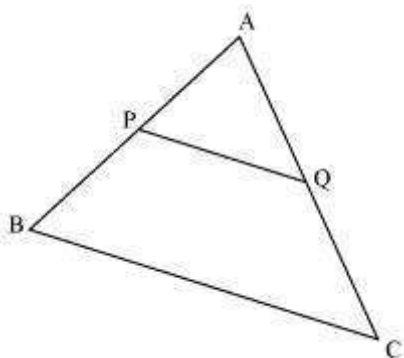
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Q8 :

Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer :



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., $AP = PB$ and $AQ = QC$

It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

$$\text{and } \frac{AQ}{QC} = \frac{1}{1}$$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

Hence, by using basic proportionality theorem, we obtain

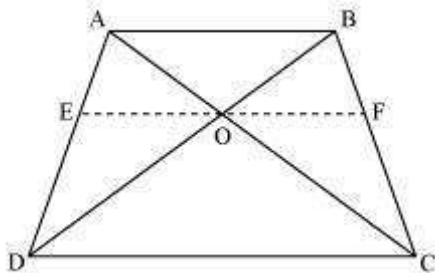
$$PQ \parallel BC$$

Q9 :

ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point **O**. Show

that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer :



Draw a line EF through point O , such that $EF \parallel CD$

In $\triangle ADC$, $EO \parallel CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \quad (1)$$

In $\triangle ABD$, $OE \parallel AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

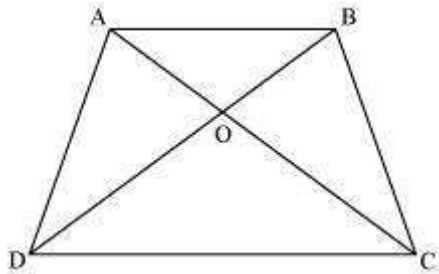
$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

Q10 :

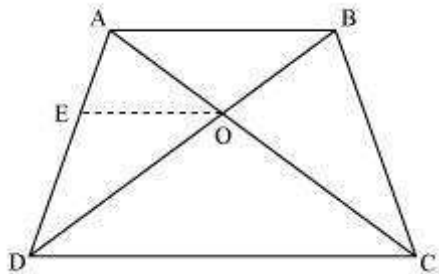
The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer :

Let us consider the following figure for the given question.



Draw a line OE || AB



In $\triangle ABD$, $OE \parallel AB$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \quad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

$\Rightarrow EO \parallel DC$ [By the converse of basic proportionality theorem]

$\Rightarrow AB \parallel OE \parallel DC$

$\Rightarrow AB \parallel CD$

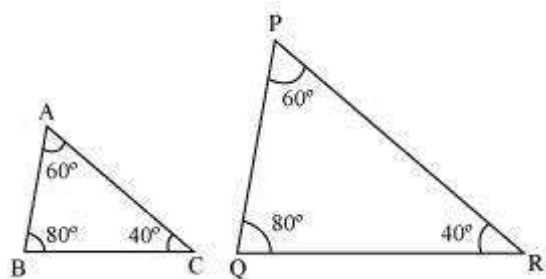
$\therefore ABCD$ is a trapezium.

Exercise 6.3 : Solutions of Questions on Page Number : 138

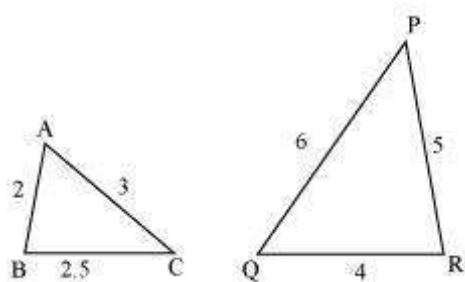
Q1 :

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

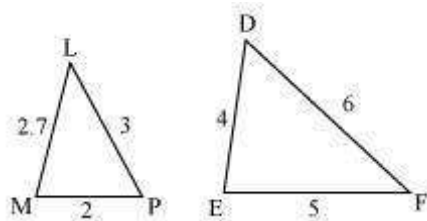
(i)



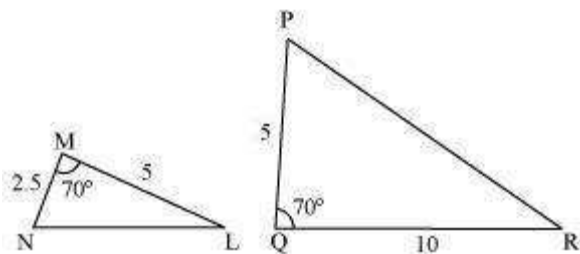
(ii)



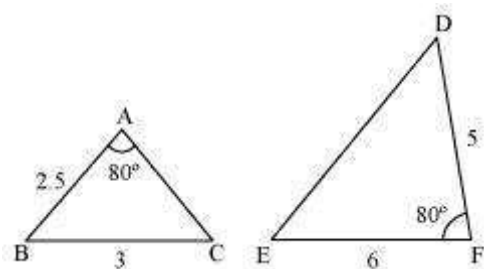
(iii)



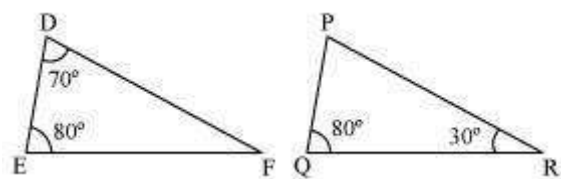
(iv)



(v)



(vi)



Answer :

(i) $\angle A = \angle P = 60^\circ$

$\angle B = \angle Q = 80^\circ$

$\angle C = \angle R = 40^\circ$

Therefore, $\triangle ABC \sim \triangle PQR$ [By AAA similarity criterion]

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii)

$\therefore \triangle ABC \sim \triangle QRP$ [By SSS similarity criterion]

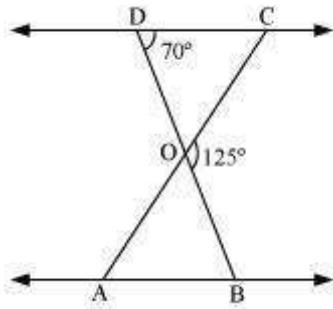
(iii) The given triangles are not similar as the corresponding sides are not proportional.

(iv) In $\triangle MNL$ and $\triangle PQR$, we observe that,

$$MN \cdot PQ = ML \cdot QR = 12$$

Q2 :

In the following figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer :

DOB is a straight line.

$$\therefore \angle DOC + \angle COB = 180^\circ$$

$$\Rightarrow \angle DOC = 180^\circ - 125^\circ$$

$$= 55^\circ$$

In $\triangle DOC$,

$$\angle DCO + \angle CDO + \angle DOC = 180^\circ$$

(Sum of the measures of the angles of a triangle is 180° .)

$$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$$

$$\Rightarrow \angle DCO = 55^\circ$$

It is given that $\triangle ODC \sim \triangle OBA$.

$$\therefore \angle OAB = \angle OCD \text{ [Corresponding angles are equal in similar triangles.]}$$

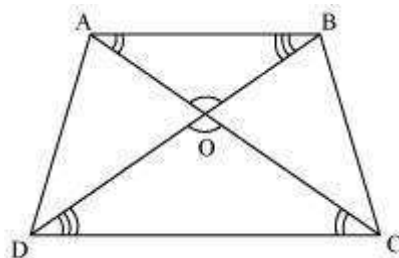
$$\Rightarrow \angle OAB = 55^\circ$$

Q3 :

Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using a

similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer :



In $\triangle ODC$ and $\triangle OBA$,

$\angle CDO = \angle ABO$ [Alternate interior angles as $AB \parallel CD$]

$\angle DCO = \angle BAO$ [Alternate interior angles as $AB \parallel CD$]

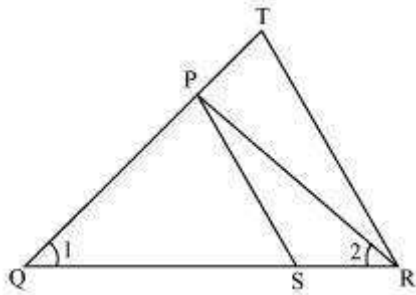
$\angle DOC = \angle BOA$ [Vertically opposite angles]

$\therefore \triangle DOC \sim \triangle BOA$ [AAA similarity criterion]

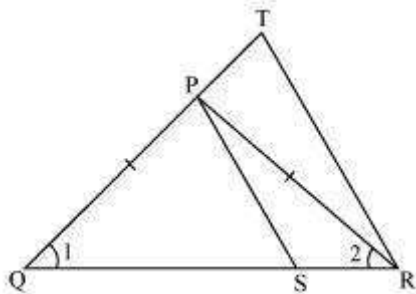
$$\begin{aligned}\therefore \frac{DO}{BO} &= \frac{OC}{OA} && \text{[Corresponding sides are proportional]} \\ \Rightarrow \frac{OA}{OC} &= \frac{OB}{OD}\end{aligned}$$

Q4 :

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\triangle PQS \sim \triangle TQR$



Answer :



In $\triangle PQR$, $\angle PQR = \angle PRQ$

$\therefore PQ = PR$ (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \quad (ii)$$

In ΔPQS and ΔTQR ,

$$\frac{QR}{QS} = \frac{QT}{QP} \quad [\text{Using (ii)}]$$

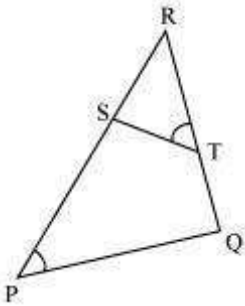
$$\angle Q = \angle Q$$

$$\therefore \Delta PQS \sim \Delta TQR \quad [\text{SAS similarity criterion}]$$

Q5 :

S and T are point on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Answer :



In ΔRPQ and ΔRTS ,

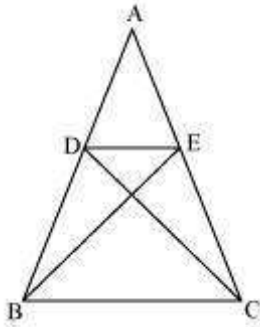
$$\angle RTS = \angle QPS \text{ (Given)}$$

$$\angle R = \angle R \text{ (Common angle)}$$

$$\therefore \Delta RPQ \sim \Delta RTS \text{ (By AA similarity criterion)}$$

Q6 :

In the following figure, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



Answer :

It is given that $\triangle ABE \cong \triangle ACD$.

$\therefore AB = AC$ [By CPCT] (1)

And, $AD = AE$ [By CPCT] (2)

In $\triangle ADE$ and $\triangle ABC$,

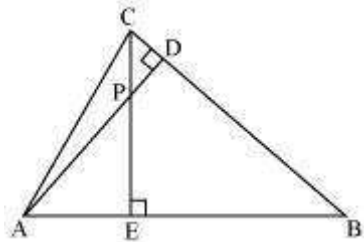
$$\frac{AD}{AB} = \frac{AE}{AC} \quad [\text{Dividing equation (2) by (1)}]$$

$\angle A = \angle A$ [Common angle]

$\therefore \triangle ADE \sim \triangle ABC$ [By SAS similarity criterion]

Q7 :

In the following figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that:



(i) $\triangle AEP \sim \triangle CDP$

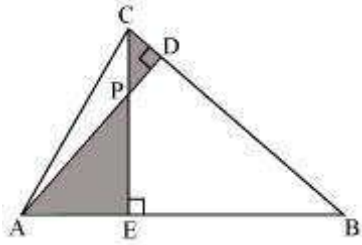
(ii) $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$

(v) $\triangle PDC \sim \triangle BEC$

Answer :

(i)



In $\triangle AEP$ and $\triangle CDP$,

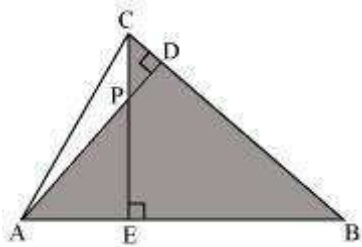
$$\angle AEP = \angle CDP \text{ (Each } 90^\circ)$$

$$\angle APE = \angle CPD \text{ (Vertically opposite angles)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle CDP$$

(ii)



In $\triangle ABD$ and $\triangle CBE$,

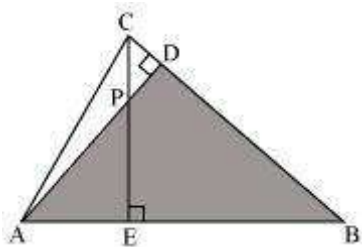
$$\angle ADB = \angle CEB \text{ (Each } 90^\circ)$$

$$\angle ABD = \angle CBE \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle ABD \sim \triangle CBE$$

(iii)



In $\triangle AEP$ and $\triangle ADB$,

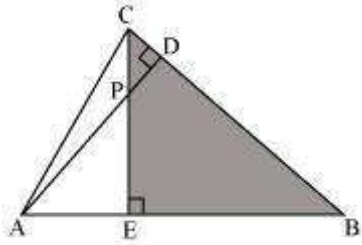
$$\angle AEP = \angle ADB \text{ (Each } 90^\circ)$$

$$\angle PAE = \angle DAB \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle ADB$$

(iv)



In $\triangle PDC$ and $\triangle BEC$,

$\angle PDC = \angle BEC$ (Each 90°)

$\angle PCD = \angle BCE$ (Common angle)

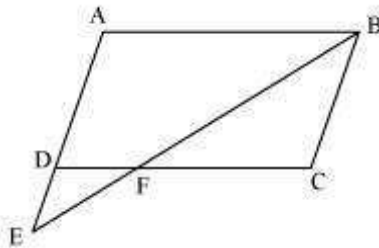
Hence, by using AA similarity criterion,

$\triangle PDC \sim \triangle BEC$

Q8 :

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$

Answer :



In $\triangle ABE$ and $\triangle CFB$,

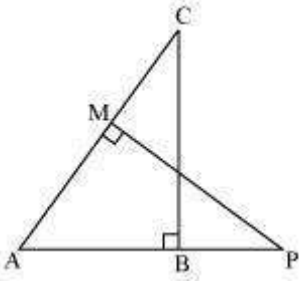
$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles as $AE \parallel BC$)

$\therefore \triangle ABE \sim \triangle CFB$ (By AA similarity criterion)

Q9 :

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) $\Delta ABC \sim \Delta AMP$

$$\frac{CA}{PA} = \frac{BC}{MP}$$

(ii)

Answer :

In ΔABC and ΔAMP ,

$\angle ABC = \angle AMP$ (Each 90°)

$\angle A = \angle A$ (Common)

$\therefore \Delta ABC \sim \Delta AMP$ (By AA similarity criterion)

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP} \quad (\text{Corresponding sides of similar triangles are proportional})$$

Q10 :

CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, Show that:

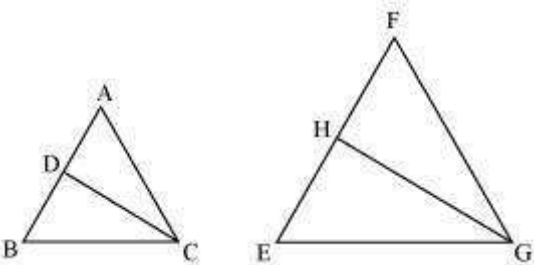
$$\frac{CD}{GH} = \frac{AC}{FG}$$

(i)

(ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Answer :



It is given that $\Delta ABC \sim \Delta FEG$.

$\therefore \angle A = \angle F$, $\angle B = \angle E$, and $\angle ACB = \angle FGE$

$$\angle ACB = \angle FGE$$

$$\therefore \angle ACD = \angle FGH \text{ (Angle bisector)}$$

$$\text{And, } \angle DCB = \angle HGE \text{ (Angle bisector)}$$

In $\triangle ACD$ and $\triangle FGH$,

$$\angle A = \angle F \text{ (Proved above)}$$

$$\angle ACD = \angle FGH \text{ (Proved above)}$$

$$\therefore \triangle ACD \sim \triangle FGH \text{ (By AA similarity criterion)}$$

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In $\triangle DCB$ and $\triangle HGE$,

$$\angle DCB = \angle HGE \text{ (Proved above)}$$

$$\angle B = \angle E \text{ (Proved above)}$$

$$\therefore \triangle DCB \sim \triangle HGE \text{ (By AA similarity criterion)}$$

In $\triangle DCA$ and $\triangle HGF$,

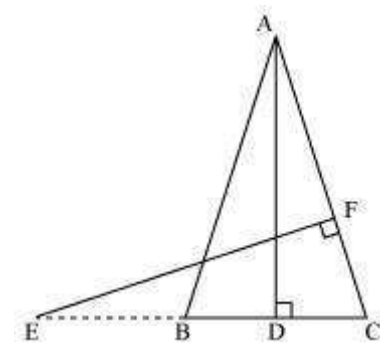
$$\angle ACD = \angle FGH \text{ (Proved above)}$$

$$\angle A = \angle F \text{ (Proved above)}$$

$$\therefore \triangle DCA \sim \triangle HGF \text{ (By AA similarity criterion)}$$

Q11 :

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$



Answer :

It is given that ABC is an isosceles triangle.

$$\therefore AB = AC$$

$$\Rightarrow \angle ABD = \angle ECF$$

In $\triangle ABD$ and $\triangle ECF$,

$$\angle ADB = \angle EFC \text{ (Each } 90^\circ \text{)}$$

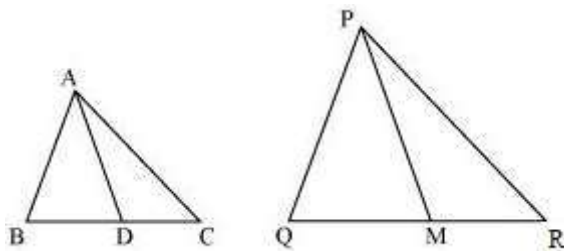
$$\angle BAD = \angle CEF \text{ (Proved above)}$$

$$\therefore \triangle ABD \sim \triangle ECF \text{ (By using AA similarity criterion)}$$

Q12 :

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see the given figure). Show that $\triangle ABC \sim \triangle PQR$.

Answer :



Median divides the opposite side.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2}$$

Given that,

$$\begin{aligned} \frac{AB}{PQ} &= \frac{BC}{QR} = \frac{AD}{PM} \\ \Rightarrow \frac{AB}{PQ} &= \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM} \\ \Rightarrow \frac{AB}{PQ} &= \frac{BD}{QM} = \frac{AD}{PM} \end{aligned}$$

In $\triangle ABD$ and $\triangle PQM$,

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \text{ (Proved above)}$$

$$\therefore \triangle ABD \sim \triangle PQM \text{ (By SSS similarity criterion)}$$

$$\Rightarrow \angle ABD = \angle PQM \text{ (Corresponding angles of similar triangles)}$$

In $\triangle ABC$ and $\triangle PQR$,

$$\angle ABD = \angle PQM \text{ (Proved above)}$$

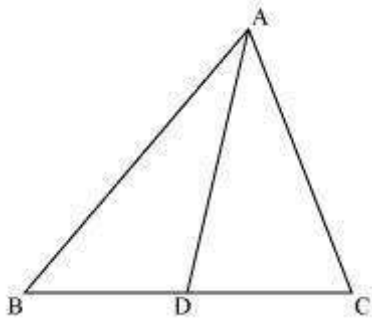
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

Q13 :

D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.

Answer :



In $\triangle ADC$ and $\triangle BAC$,

$\angle ADC = \angle BAC$ (Given)

$\angle ACD = \angle BCA$ (Common angle)

$\therefore \triangle ADC \sim \triangle BAC$ (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

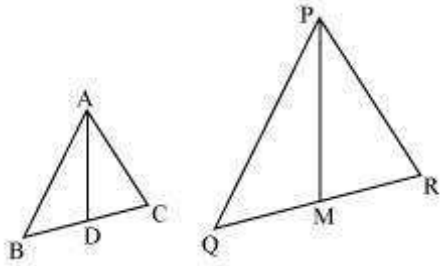
$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB \times CD$$

Q14 :

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$

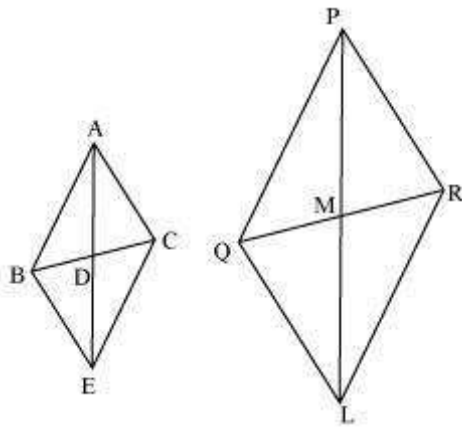
Answer :



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

$\therefore$ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

$\therefore \triangle ABE \sim \triangle PQL$ (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

$$\therefore \angle BAE = \angle QPL \dots (1)$$

Similarly, it can be proved that $\triangle AEC \sim \triangle PLR$ and

$$\angle CAE = \angle RPL \dots (2)$$

Adding equation (1) and (2), we obtain

$$\angle BAE + \angle CAE = \angle QPL + \angle RPL$$

$$\Rightarrow \angle CAB = \angle RPQ \dots (3)$$

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad (\text{Given})$$

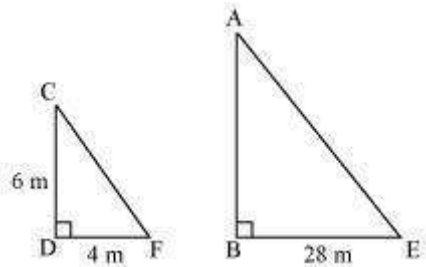
$$\angle CAB = \angle RPQ \text{ [Using equation (3)]}$$

$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

Q15 :

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer :



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, $\angle DCF = \angle BAE$

And, $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)

$\therefore \triangle ABE \sim \triangle CDF$ (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6 \text{ m}} = \frac{28}{4}$$

$$\Rightarrow AB = 42 \text{ m}$$

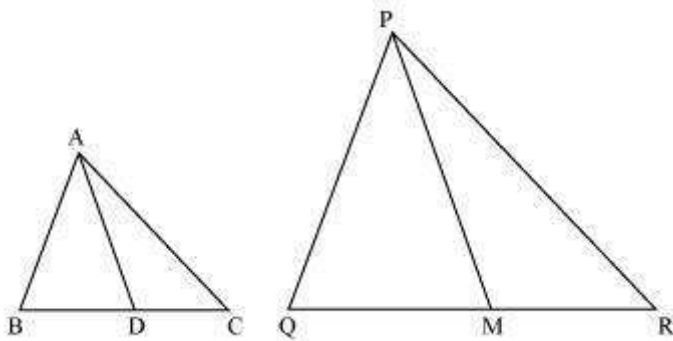
Therefore, the height of the tower will be 42 metres.

Q16 :

If AD and PM are medians of triangles ABC and PQR, respectively

$\Delta ABC \sim \Delta PQR$ prove that $\frac{AB}{PQ} = \frac{AD}{PM}$
where

Answer :



It is given that $\Delta ABC \sim \Delta PQR$

We know that the corresponding sides of similar triangles are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \dots (1)$$

Also, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R \dots (2)$

Since AD and PM are medians, they will divide their opposite sides.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2} \dots (3)$$

From equations (1) and (3), we obtain

$$\frac{AB}{PQ} = \frac{BD}{QM} \dots (4)$$

In ΔABD and ΔPQM ,

$\angle B = \angle Q$ [Using equation (2)]

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad [\text{Using equation (4)}]$$

$\therefore \triangle ABD \sim \triangle PQM$ (By SAS similarity criterion)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

Exercise 6.4 : Solutions of Questions on Page Number : 143

Q1 :

Let $\triangle ABC \sim \triangle DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2 . If $EF = 15.4 \text{ cm}$, find BC .

Answer :

It is given that $\triangle ABC \sim \triangle DEF$.

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$$EF = 15.4 \text{ cm},$$

$$\text{ar}(\triangle ABC) = 64 \text{ cm}^2,$$

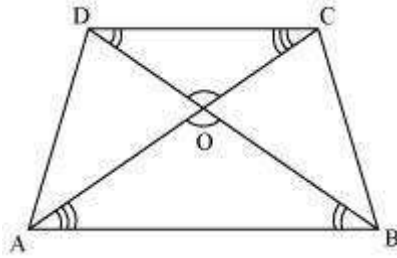
$$\text{ar}(\triangle DEF) = 121 \text{ cm}^2$$

$$\begin{aligned} \therefore \frac{\text{ar}(ABC)}{\text{ar}(DEF)} &= \left(\frac{BC}{EF}\right)^2 \\ \Rightarrow \left(\frac{64 \text{ cm}^2}{121 \text{ cm}^2}\right) &= \frac{BC^2}{(15.4 \text{ cm})^2} \\ \Rightarrow \frac{BC}{15.4} &= \left(\frac{8}{11}\right) \text{ cm} \\ \Rightarrow BC &= \left(\frac{8 \times 15.4}{11}\right) \text{ cm} = (8 \times 1.4) \text{ cm} = 11.2 \text{ cm} \end{aligned}$$

Q2 :

Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O . If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD .

Answer :



Since $AB \parallel CD$,

$\therefore \angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$ (Alternate interior angles)

In $\triangle AOB$ and $\triangle COD$,

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\angle OBA = \angle ODC$ (Alternate interior angles)

$\therefore \triangle AOB \sim \triangle COD$ (By AAA similarity criterion)

$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{AB}{CD}\right)^2$$

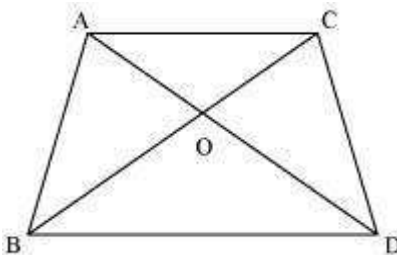
Since $AB = 2 CD$,

$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{2 CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

Q3 :

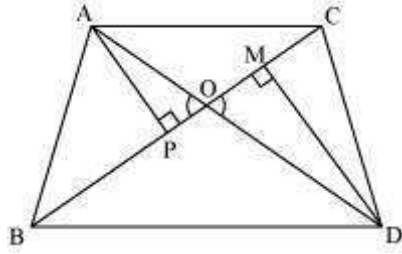
In the following figure, ABC and DBC are two triangles on the same base BC . If AD intersects BC at O , show

that $\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$



Answer :

Let us draw two perpendiculars AP and DM on line BC .



We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$$

In $\triangle APO$ and $\triangle DMO$,

$\angle APO = \angle DMO$ (Each = 90°)

$\angle AOP = \angle DOM$ (Vertically opposite angles)

$\therefore \triangle APO \sim \triangle DMO$ (By AA similarity criterion)

$$\therefore \frac{AP}{DM} = \frac{AO}{DO}$$

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$$

Q4 :

If the areas of two similar triangles are equal, prove that they are congruent.

Answer :

Let us assume two similar triangles as $\triangle ABC \sim \triangle PQR$.

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \quad (1)$$

Given that, $\text{ar}(\Delta ABC) = \text{ar}(\Delta PQR)$

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

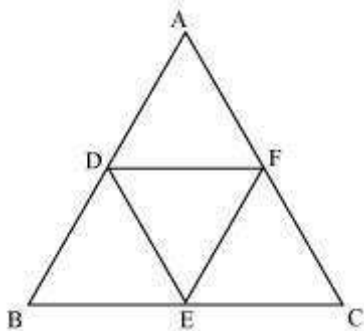
$\Rightarrow AB = PQ, BC = QR, \text{ and } AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ (By SSS congruence criterion)

Q5 :

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC . Find the ratio of the area of ΔDEF and ΔABC .

Answer :



D and E are the mid-points of ΔABC .

$$\therefore DE \parallel AC \text{ and } DE = \frac{1}{2} AC$$

In $\triangle BED$ and $\triangle BCA$,

$$\angle BED = \angle BCA \quad (\text{Corresponding angles})$$

$$\angle BDE = \angle BAC \quad (\text{Corresponding angles})$$

$$\angle EBD = \angle CBA \quad (\text{Common angles})$$

$$\therefore \triangle BED \sim \triangle BCA \quad (\text{AAA similarity criterion})$$

$$\frac{\text{ar}(\triangle BED)}{\text{ar}(\triangle BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{\text{ar}(\triangle BED)}{\text{ar}(\triangle BCA)} = \frac{1}{4}$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle BCA)$$

$$\text{Similarly, } \text{ar}(\triangle CFE) = \frac{1}{4} \text{ar}(\triangle CBA) \text{ and } \text{ar}(\triangle ADF) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\text{Also, } \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC) - [\text{ar}(\triangle BED) + \text{ar}(\triangle CFE) + \text{ar}(\triangle ADF)]$$

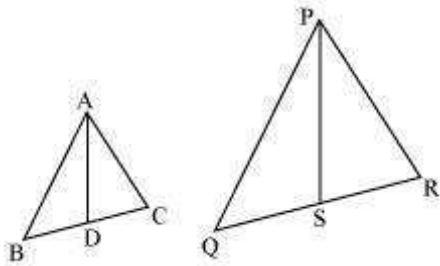
$$\Rightarrow \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC) - \frac{3}{4} \text{ar}(\triangle ABC) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\Rightarrow \frac{\text{ar}(\triangle DEF)}{\text{ar}(\triangle ABC)} = \frac{1}{4}$$

Q6 :

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer :



Let us assume two similar triangles as $\triangle ABC \sim \triangle PQR$. Let AD and PS be the medians of these triangles.

$$\therefore \triangle ABC \sim \triangle PQR$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \dots (1)$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots (2)$$

Since AD and PS are medians,

$$\therefore BD = DC = \frac{BC}{2}$$

$$\text{And, } QS = SR = \frac{QR}{2}$$

Equation (1) becomes

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR} \dots (3)$$

In $\triangle ABD$ and $\triangle PQS$,

$$\angle B = \angle Q \text{ [Using equation (2)]}$$

$$\text{And, } \frac{AB}{PQ} = \frac{BD}{QS} \text{ [Using equation (3)]}$$

$$\therefore \triangle ABD \sim \triangle PQS \text{ (SAS similarity criterion)}$$

Therefore, it can be said that

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS} \dots (4)$$

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

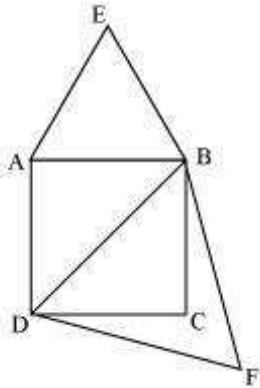
And hence,

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AD}{PS}\right)^2$$

Q7 :

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer :



Let ABCD be a square of side a .

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as $\triangle ABE$ and $\triangle DBF$.

Side of an equilateral triangle, $\triangle ABE$, described on one of its sides $= a$

Side of an equilateral triangle, $\triangle DBF$, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \triangle ABE}{\text{Area of } \triangle DBF} = \left(\frac{a}{\sqrt{2}a} \right)^2 = \frac{1}{2}$$

Q8 :

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

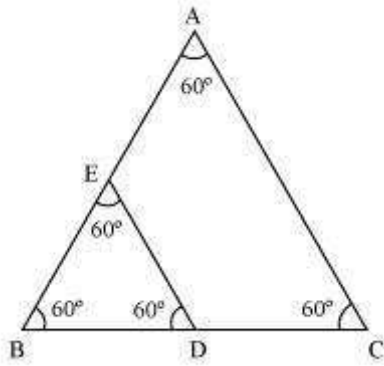
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

Answer :



We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of $\triangle ABC = x$

Therefore, side of $\triangle BDE = \frac{x}{2}$

$$\therefore \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle BDE)} = \left(\frac{x}{\frac{x}{2}} \right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Q9 :

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

Answer :

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9} \right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

Exercise 6.5 : Solutions of Questions on Page Number : 150

Q1 :

Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Answer :

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

$$49 + 576 = 625$$

$$\text{Or, } 7^2 + 24^2 = 25^2$$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

$$\text{However, } 9 + 36 \neq 64$$

$$\text{Or, } 3^2 + 6^2 \neq 8^2$$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii) Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

$$\text{However, } 2500 + 6400 \neq 10000$$

$$\text{Or, } 50^2 + 80^2 \neq 100^2$$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv) Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly, $144 + 25 = 169$

Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

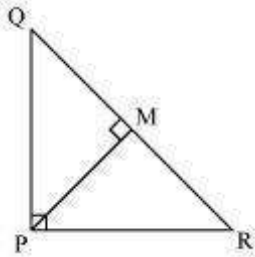
We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

Q2 :

PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM \times MR$.

Answer :



Let $\angle MPR = x$

In $\triangle MPR$,

$$\angle MRP = 180^\circ - 90^\circ - x$$

$$\angle MRP = 90^\circ - x$$

Similarly, in $\triangle MPQ$,

$$\angle MPQ = 90^\circ - \angle MPR$$

$$= 90^\circ - x$$

$$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x)$$

$$\angle MQP = x$$

In $\triangle QMP$ and $\triangle PMR$,

$$\angle MPQ = \angle MRP$$

$$\angle PMQ = \angle RMP$$

$$\angle MQP = \angle MPR$$

$\therefore \triangle QMP \sim \triangle PMR$ (By AAA similarity criterion)

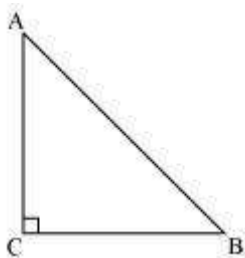
$$\Rightarrow \frac{QM}{PM} = \frac{MP}{MR}$$

$$\Rightarrow PM^2 = QM \times MR$$

Q3 :

ABC is an isosceles triangle right angled at C. prove that $AB^2 = 2 AC^2$.

Answer :



Given that $\triangle ABC$ is an isosceles triangle.

$$\therefore AC = CB$$

Applying Pythagoras theorem in $\triangle ABC$ (i.e., right-angled at point C), we obtain

$$AC^2 + CB^2 = AB^2$$

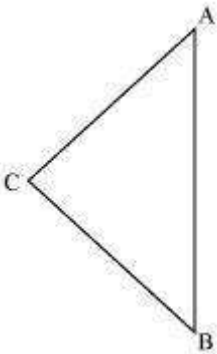
$$\Rightarrow AC^2 + AC^2 = AB^2 \quad (AC = CB)$$

$$\Rightarrow 2AC^2 = AB^2$$

Q4 :

ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2 AC^2$, prove that ABC is a right triangle.

Answer :



Given that,

$$AB^2 = 2AC^2$$

$$\Rightarrow AB^2 = AC^2 + AC^2$$

$$\Rightarrow AB^2 = AC^2 + BC^2 \quad (\text{As } AC = BC)$$

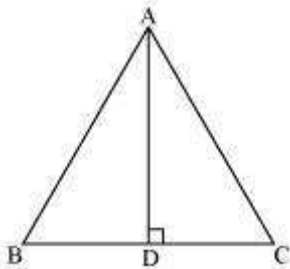
The triangle is satisfying the pythagoras theorem.

Therefore, the given triangle is a right - angled triangle.

Q5 :

ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Answer :



Let AD be the altitude in the given equilateral triangle, ΔABC .

We know that altitude bisects the opposite side.

$$\therefore BD = DC = a$$

In $\triangle ADB$,

$$\angle ADB = 90^\circ$$

Applying pythagoras theorem, we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 + a^2 = (2a)^2$$

$$\Rightarrow AD^2 + a^2 = 4a^2$$

$$\Rightarrow AD^2 = 3a^2$$

$$\Rightarrow AD = a\sqrt{3}$$

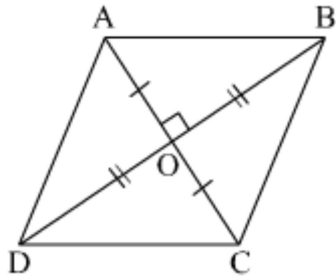
In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be $\sqrt{3}a$.

Q6 :

Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Answer :



In $\triangle AOB$, $\triangle BOC$, $\triangle COD$, $\triangle AOD$,

Applying Pythagoras theorem, we obtain

$$AB^2 = AO^2 + OB^2 \quad \dots (1)$$

$$BC^2 = BO^2 + OC^2 \quad \dots (2)$$

$$CD^2 = CO^2 + OD^2 \quad \dots (3)$$

$$AD^2 = AO^2 + OD^2 \quad \dots (4)$$

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2(AO^2 + OB^2 + OC^2 + OD^2)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

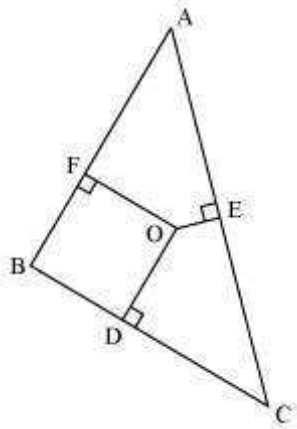
(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

$$= (AC)^2 + (BD)^2$$

Q7 :

In the following figure, O is a point in the interior of a triangle ABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that

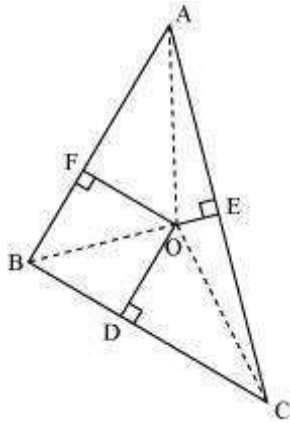


(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer :

Join OA, OB, and OC.



(i) Applying Pythagoras theorem in $\triangle AOF$, we obtain

$$OA^2 = OF^2 + AF^2$$

Similarly, in $\triangle BOD$,

$$OB^2 = OD^2 + BD^2$$

Similarly, in $\triangle COE$,

$$OC^2 = OE^2 + EC^2$$

Adding these equations,

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$$

(ii) From the above result,

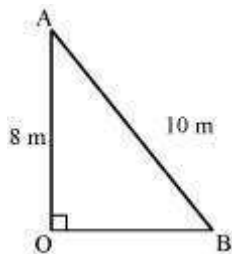
$$AF^2 + BD^2 + EC^2 = (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$$

$$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$$

Q8 :

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer :



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

$$AB^2 = OA^2 + BO^2$$

$$(10 \text{ m})^2 = (8 \text{ m})^2 + OB^2$$

$$100 \text{ m}^2 = 64 \text{ m}^2 + OB^2$$

$$OB^2 = 36 \text{ m}^2$$

$$OB = 6 \text{ m}$$

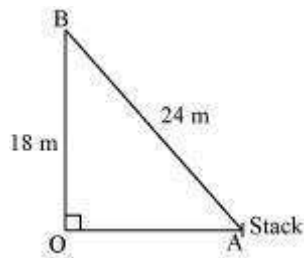
Therefore, the distance of the foot of the ladder from the base of the wall is

6 m.

Q9 :

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer :



Let OB be the pole and AB be the wire.

By Pythagoras theorem,

$$AB^2 = OB^2 + OA^2$$

$$(24 \text{ m})^2 = (18 \text{ m})^2 + OA^2$$

$$OA^2 = (576 - 324) \text{ m}^2 = 252 \text{ m}^2$$

$$OA = \sqrt{252} \text{ m} = \sqrt{6 \times 6 \times 7} \text{ m} = 6\sqrt{7} \text{ m}$$

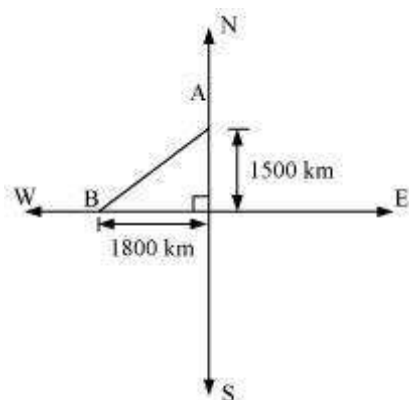
Therefore, the distance from the base is $6\sqrt{7}$ m.

Q10 :

An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be

the two planes after $1\frac{1}{2}$ hours?

Answer :



Distance travelled by the plane flying towards north in $1\frac{1}{2}$ hrs $= 1,000 \times 1\frac{1}{2} = 1,500$ km

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}$ hrs $= 1,200 \times 1\frac{1}{2} = 1,800$ km

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

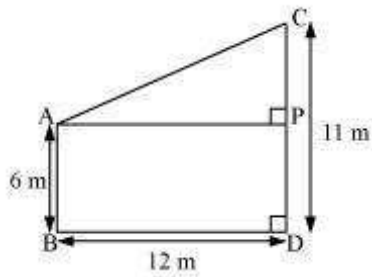
$$\begin{aligned} \text{Distance between these planes after } 1\frac{1}{2} \text{ hrs, } AB &= \sqrt{OA^2 + OB^2} \\ &= \left(\sqrt{(1,500)^2 + (1,800)^2} \right) \text{ km} = \left(\sqrt{2250000 + 3240000} \right) \text{ km} \\ &= \left(\sqrt{5490000} \right) \text{ km} = \left(\sqrt{9 \times 610000} \right) \text{ km} = 300\sqrt{61} \text{ km} \end{aligned}$$

Therefore, the distance between these planes will be $300\sqrt{61}$ km after $1\frac{1}{2}$ hrs.

Q11 :

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer :



Let CD and AB be the poles of height 11 m and 6 m.

Therefore, $CP = 11 - 6 = 5$ m

From the figure, it can be observed that $AP = 12$ m

Applying Pythagoras theorem for $\triangle APC$, we obtain

$$AP^2 + PC^2 = AC^2$$

$$(12 \text{ m})^2 + (5 \text{ m})^2 = AC^2$$

$$AC^2 = (144 + 25) \text{ m}^2 = 169 \text{ m}^2$$

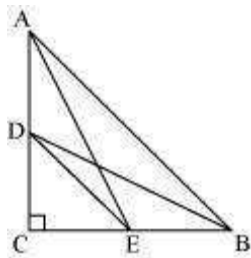
$$AC = 13 \text{ m}$$

Therefore, the distance between their tops is 13 m.

Q12 :

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$

Answer :



Applying Pythagoras theorem in $\triangle ACE$, we obtain

$$AC^2 + CE^2 = AE^2 \quad \dots (1)$$

Applying Pythagoras theorem in $\triangle BCD$, we obtain

$$BC^2 + CD^2 = BD^2 \quad \dots (2)$$

Using equation (1) and equation (2), we obtain

$$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \quad \dots (3)$$

Applying Pythagoras theorem in $\triangle CDE$, we obtain

$$DE^2 = CD^2 + CE^2$$

Applying Pythagoras theorem in $\triangle ABC$, we obtain

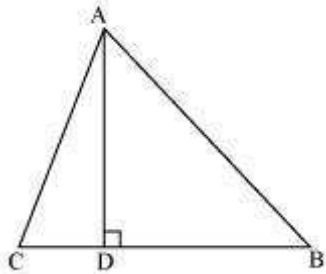
$$AB^2 = AC^2 + CB^2$$

Putting the values in equation (3), we obtain

$$DE^2 + AB^2 = AE^2 + BD^2$$

Q13 :

The perpendicular from A on side BC of a $\triangle ABC$ intersect BC at D such that $DB = 3 CD$. Prove that $2 AB^2 = 2 AC^2 + BC^2$



Answer :

Applying Pythagoras theorem for $\triangle ACD$, we obtain

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \quad \dots (1)$$

Applying Pythagoras theorem in $\triangle ABD$, we obtain

$$AB^2 = AD^2 + DB^2$$

$$AD^2 = AB^2 - DB^2 \quad \dots (2)$$

From equation (1) and equation (2), we obtain

$$AC^2 - DC^2 = AB^2 - DB^2 \quad \dots (3)$$

It is given that $3DC = DB$

$$\therefore DC = \frac{BC}{4} \text{ and } DB = \frac{3BC}{4}$$

Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

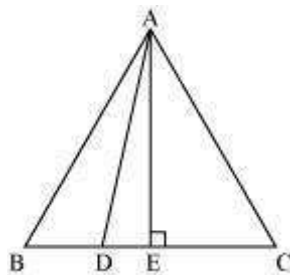
$$16AB^2 - 16AC^2 = 8BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

Q14 :

In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3} BC$. Prove that $9 AD^2 = 7 AB^2$.

Answer :



Let the side of the equilateral triangle be a , and AE be the altitude of $\triangle ABC$.

$$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$$

$$\text{And, } AE = \frac{a\sqrt{3}}{2}$$

$$\text{Given that, } BD = \frac{1}{3} BC$$

$$\therefore BD = \frac{a}{3}$$

$$DE = BE - BD = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$$

Applying Pythagoras theorem in $\triangle ADE$, we obtain

$$AD^2 = AE^2 + DE^2$$

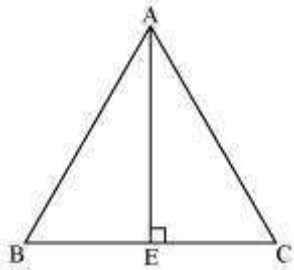
$$\begin{aligned} AD^2 &= \left(\frac{a\sqrt{3}}{2} \right)^2 + \left(\frac{a}{6} \right)^2 \\ &= \left(\frac{3a^2}{4} \right) + \left(\frac{a^2}{36} \right) \\ &= \frac{28a^2}{36} \\ &= \frac{7}{9} AB^2 \end{aligned}$$

$$\Rightarrow 9 AD^2 = 7 AB^2$$

Q15 :

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer :



Let the side of the equilateral triangle be a , and AE be the altitude of $\triangle ABC$.

$$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$$

Applying Pythagoras theorem in $\triangle ABE$, we obtain

$$AB^2 = AE^2 + BE^2$$

$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

$$4AE^2 = 3a^2$$

$\Rightarrow 4 \times (\text{Square of altitude}) = 3 \times (\text{Square of one side})$

Q16 :

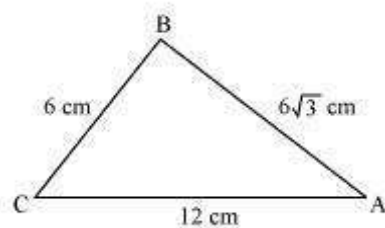
Tick the correct answer and justify: In $\triangle ABC$, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm.

The angle B is:

(A) 120° (B) 60°

(C) 90° (D) 45°

Answer :



Given that, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm, and $BC = 6$ cm

It can be observed that

$$AB^2 = 108$$

$$AC^2 = 144$$

$$\text{And, } BC^2 = 36$$

$$AB^2 + BC^2 = AC^2$$

The given triangle, $\triangle ABC$, is satisfying Pythagoras theorem.

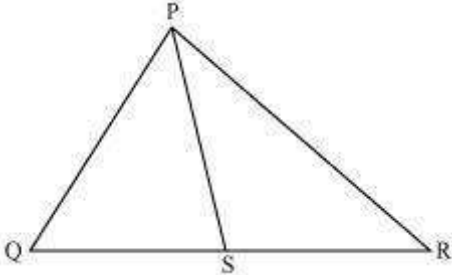
Therefore, the triangle is a right triangle, right-angled at B.

$$\therefore \angle B = 90^\circ$$

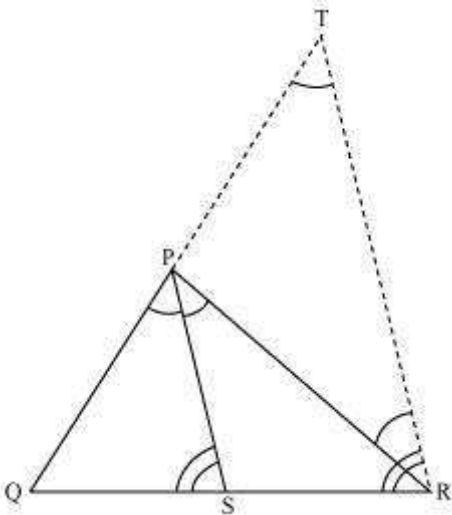
Hence, the correct answer is (C).

Q1 :

In the given figure, PS is the bisector of $\angle QPR$ of ΔPQR . Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$.



Answer :



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of $\angle QPR$.

$$\angle QPS = \angle SPR \dots (1)$$

By construction,

$$\angle SPR = \angle PRT \text{ (As } PS \parallel TR \text{)} \dots (2)$$

$$\angle QPS = \angle QTR \text{ (As } PS \parallel TR \text{)} \dots (3)$$

Using these equations, we obtain

$$\angle PRT = \angle QTR$$

$$\therefore PT = PR$$

By construction,

$$PS \parallel TR$$

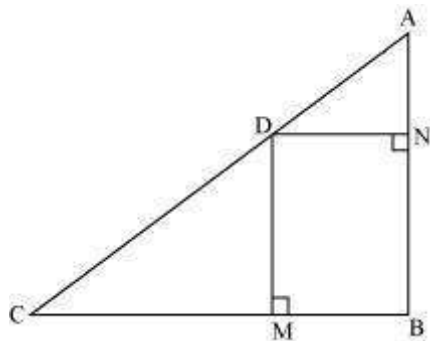
By using basic proportionality theorem for ΔQTR ,
 $QSSR = QPPT$
 $\Rightarrow QSSR$

Q2 :

In the given figure, D is a point on hypotenuse AC of ΔABC , $DM \perp BC$ and $DN \perp AB$, Prove that:

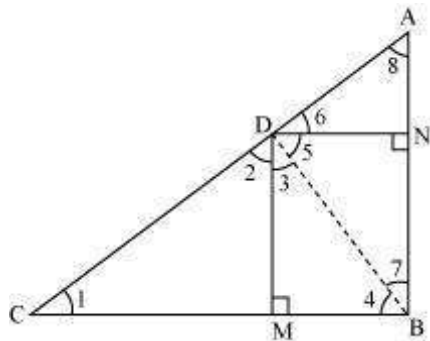
(i) $DM^2 = DN \cdot MC$

(ii) $DN^2 = DM \cdot AN$



Answer :

(i) Let us join DB.



We have, $DN \parallel CB$, $DM \parallel AB$, and $\angle B = 90^\circ$

$\therefore$ DMBN is a rectangle.

$\therefore DN = MB$ and $DM = NB$

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle CDB = 90^\circ$

$$\Rightarrow \angle 2 + \angle 3 = 90^\circ \dots (1)$$

In ΔCDM ,

$$\angle 1 + \angle 2 + \angle DMC = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ \dots (2)$$

In $\triangle DMB$,

$$\angle 3 + \angle DMB + \angle 4 = 180^\circ$$

$$\Rightarrow \angle 3 + \angle 4 = 90^\circ \dots (3)$$

From equation (1) and (2), we obtain

$$\angle 1 = \angle 3$$

From equation (1) and (3), we obtain

$$\angle 2 = \angle 4$$

In $\triangle DCM$ and $\triangle BDM$,

$$\angle 1 = \angle 3 \text{ (Proved above)}$$

$$\angle 2 = \angle 4 \text{ (Proved above)}$$

$\therefore \triangle DCM \sim \triangle BDM$ (AA similarity criterion)

$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$$
$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \quad (BM = DN)$$

$$\Rightarrow DM^2 = DN \times MC$$

(ii) In right triangle DBN ,

$$\angle 5 + \angle 7 = 90^\circ \dots (4)$$

In right triangle DAN ,

$$\angle 6 + \angle 8 = 90^\circ \dots (5)$$

D is the foot of the perpendicular drawn from B to AC.

$$\therefore \angle ADB = 90^\circ$$

$$\Rightarrow \angle 5 + \angle 6 = 90^\circ \dots (6)$$

From equation (4) and (6), we obtain

$$\angle 6 = \angle 7$$

From equation (5) and (6), we obtain

$$\angle 8 = \angle 5$$

In $\triangle DNA$ and $\triangle BND$,

$$\angle 6 = \angle 7 \text{ (Proved above)}$$

$$\angle 8 = \angle 5 \text{ (Proved above)}$$

$\therefore \triangle DNA \sim \triangle BND$ (AA similarity criterion)

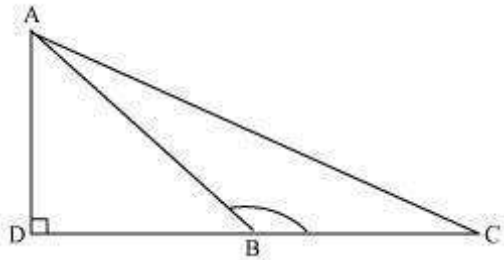
$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB}$$

$$\Rightarrow DN^2 = AN \times NB$$

$$\Rightarrow DN^2 = AN \times DM \text{ (As } NB = DM)$$

Q3 :

In the given figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and $AD \perp CB$ produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer :

Applying Pythagoras theorem in $\triangle ADB$, we obtain

$$AB^2 = AD^2 + DB^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle ACD$, we obtain

$$AC^2 = AD^2 + DC^2$$

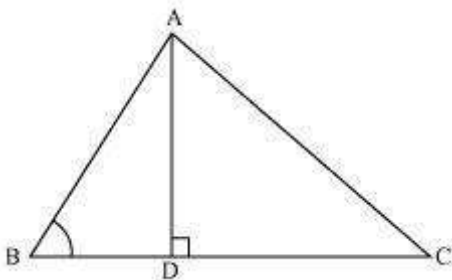
$$AC^2 = AD^2 + (DB + BC)^2$$

$$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$$

$$AC^2 = AB^2 + BC^2 + 2DB \times BC \text{ [Using equation (1)]}$$

Q4 :

In the given figure, ABC is a triangle in which $\angle ABC < 90^\circ$ and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.



Answer :

Applying Pythagoras theorem in $\triangle ADB$, we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 = AB^2 - DB^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle ADC$, we obtain

$$AD^2 + DC^2 = AC^2$$

$$AB^2 - BD^2 + DC^2 = AC^2 \text{ [Using equation (1)]}$$

$$AB^2 - BD^2 + (BC - BD)^2 = AC^2$$

$$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$$

$$= AB^2 + BC^2 - 2BC \times BD$$

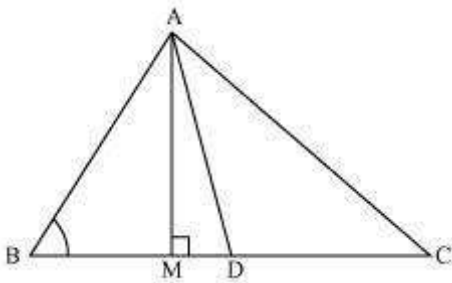
Q5 :

In the given figure, AD is a median of a triangle ABC and $AM \perp BC$. Prove that:

(i)
$$AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

(ii)
$$AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

(iii)
$$AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$$



Answer :

(i) Applying Pythagoras theorem in $\triangle AMD$, we obtain

$$AM^2 + MD^2 = AD^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle AMC$, we obtain

$$AM^2 + MC^2 = AC^2$$

$$AM^2 + (MD + DC)^2 = AC^2$$

$$(AM^2 + MD^2) + DC^2 + 2MD \cdot DC = AC^2$$

$$AD^2 + DC^2 + 2MD \cdot DC = AC^2 \text{ [Using equation (1)]}$$

Using the result,
$$DC = \frac{BC}{2},$$
 we obtain

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD \cdot \left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in $\triangle ABM$, we obtain

$$AB^2 = AM^2 + MB^2$$

$$= (AD^2 - DM^2) + MB^2$$

$$= (AD^2 - DM^2) + (BD - MD)^2$$

$$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$$

$$= AD^2 + BD^2 - 2BD \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii) Applying Pythagoras theorem in $\triangle AMC$, we obtain

$$AM^2 + MB^2 = AB^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle AMC$, we obtain

$$AM^2 + MC^2 = AC^2 \dots (2)$$

Adding equations (1) and (2), we obtain

$$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$$

$$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$$

$$2AM^2 + BD^2 + DM^2 - 2BD \cdot DM + MD^2 + DC^2 + 2MD \cdot DC = AB^2 + AC^2$$

$$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$$

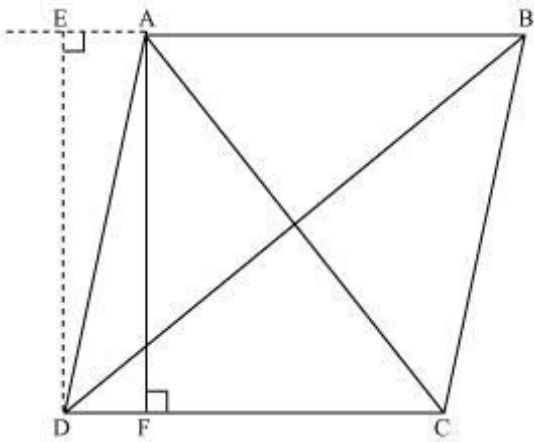
$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

Question 6:

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer:



Let ABCD be a parallelogram.

Let us draw perpendicular DE on extended side AB, and AF on side DC.

Applying Pythagoras theorem in $\triangle DEA$, we obtain

$$DE^2 + EA^2 = DA^2 \dots (i)$$

Applying Pythagoras theorem in $\triangle DEB$, we obtain

$$DE^2 + EB^2 = DB^2$$

$$DE^2 + (EA + AB)^2 = DB^2$$

$$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$$

$$DA^2 + AB^2 + 2EA \times AB = DB^2 \dots (ii)$$

Applying Pythagoras theorem in $\triangle ADF$, we obtain

$$AD^2 = AF^2 + FD^2$$

Applying Pythagoras theorem in $\triangle AFC$, we obtain

$$AC^2 = AF^2 + FC^2$$

$$= AF^2 + (DC - FD)^2$$

$$= AF^2 + DC^2 + FD^2 - 2DC \times FD$$

$$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$$

$$AC^2 = AD^2 + DC^2 - 2DC \times FD \dots (iii)$$

Since ABCD is a parallelogram,

$$AB = CD \dots (iv)$$

$$\text{And, } BC = AD \dots (v)$$

In $\triangle DEA$ and $\triangle ADF$,

$$\angle DEA = \angle AFD \text{ (Both } 90^\circ)$$

$$\angle EAD = \angle ADF \text{ (EA } \parallel \text{ DF)}$$

$$AD = AD \text{ (Common)}$$

$$\therefore \triangle EAD \cong \triangle FDA \text{ (AAS congruence criterion)}$$

$$\Rightarrow EA = DF \dots (vi)$$

Adding equations (i) and (iii), we obtain

$$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$$

$$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$$

$$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$$

[Using equations (iv) and (vi)]

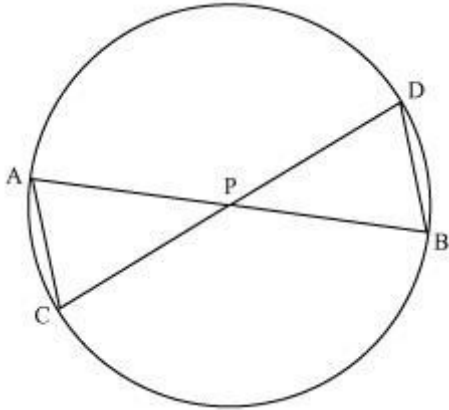
$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

Question 7:

In the given figure, two chords AB and CD intersect each other at the point P. prove that:

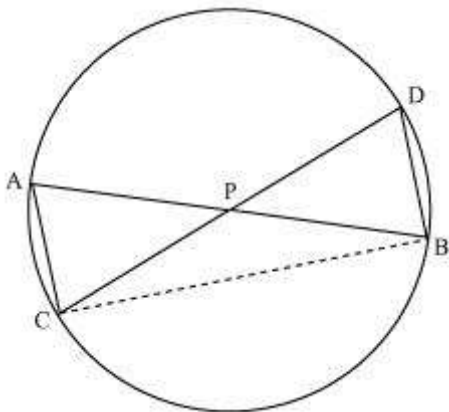
(i) $\triangle APC \sim \triangle DPB$

(ii) $AP \cdot BP = CP \cdot DP$



Answer:

Let us join CB.



(i) In $\triangle APC$ and $\triangle DPB$,

$\angle APC = \angle DPB$ (Vertically opposite angles)

$\angle CAP = \angle BDP$ (Angles in the same segment for chord CB)

$\triangle APC \sim \triangle DPB$ (By AA similarity criterion)

(ii) We have already proved that

$$\triangle APC \sim \triangle DPB$$

We know that the corresponding sides of similar triangles are proportional.

$$\begin{aligned} \therefore \frac{AP}{DP} &= \frac{PC}{PB} = \frac{CA}{BD} \\ \Rightarrow \frac{AP}{DP} &= \frac{PC}{PB} \end{aligned}$$

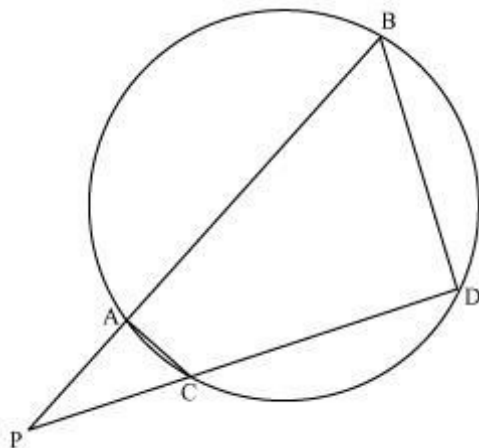
$$\therefore AP \cdot PB = PC \cdot DP$$

Question 8:

In the given figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\triangle PAC \sim \triangle PDB$

(ii) $PA \cdot PB = PC \cdot PD$



Answer:

(i) In $\triangle PAC$ and $\triangle PDB$,

$$\angle P = \angle P \text{ (Common)}$$

$\angle PAC = \angle PDB$ (Exterior angle of a cyclic quadrilateral is $\angle PCA = \angle PBD$ equal to the opposite interior angle)

$$\therefore \triangle PAC \sim \triangle PDB$$

(ii) We know that the corresponding sides of similar triangles are proportional.

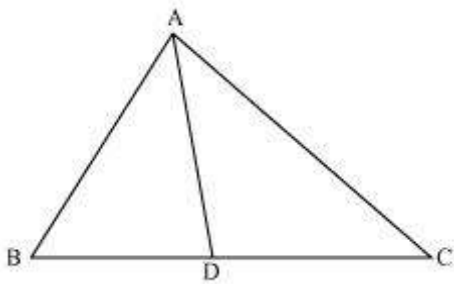
$$\therefore \frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB}$$

$$\Rightarrow \frac{PA}{PD} = \frac{PC}{PB}$$

$$\therefore PA \cdot PB = PC \cdot PD$$

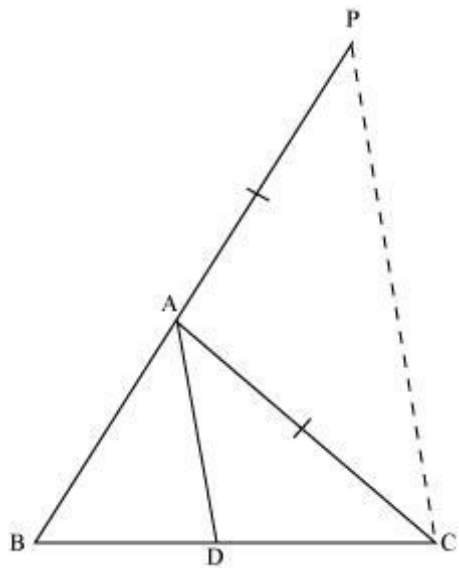
Question 9:

In the given figure, D is a point on side BC of $\triangle ABC$ such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of $\angle BAC$.



Answer:

Let us extend BA to P such that $AP = AC$. Join PC.



It is given that,

$$\frac{BD}{CD} = \frac{AB}{AC}$$

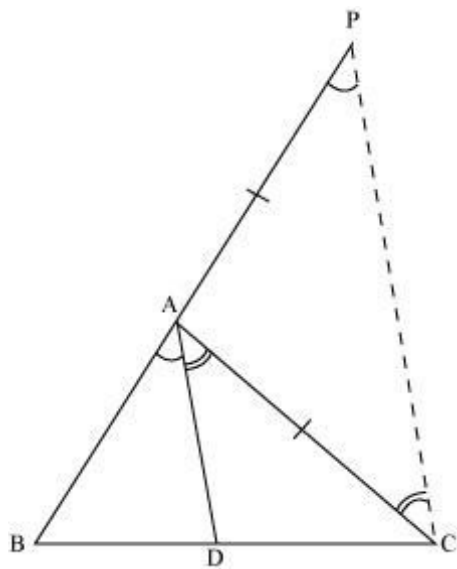
$$\Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$$

By using the converse of basic proportionality theorem, we obtain

$$AD \parallel PC$$

$$\Rightarrow \angle BAD = \angle APC \text{ (Corresponding angles) ... (1)}$$

$$\text{And, } \angle DAC = \angle ACP \text{ (Alternate interior angles) ... (2)}$$



By construction, we have

$$AP = AC$$

$$\Rightarrow \angle APC = \angle ACP \text{ ... (3)}$$

On comparing equations (1), (2), and (3), we obtain

$$\angle BAD = \angle APC$$

$$\Rightarrow AD \text{ is the bisector of the angle } BAC.$$

Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m

away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?

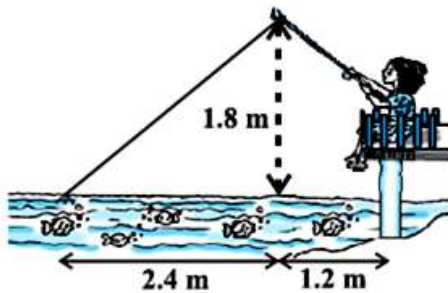
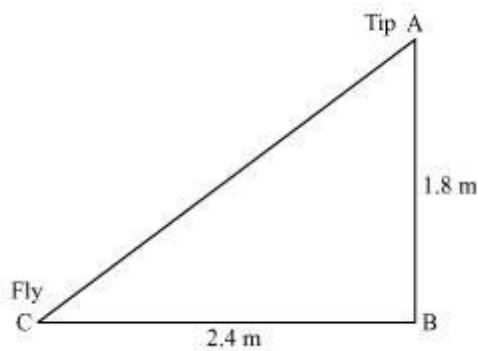


Fig. 6.64

Answer:



Let AB be the height of the tip of the fishing rod from the water surface. Let BC be the horizontal distance of the fly from the tip of the fishing rod.

Then, AC is the length of the string.

AC can be found by applying Pythagoras theorem in $\triangle ABC$.

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = (1.8 \text{ m})^2 + (2.4 \text{ m})^2$$

$$AB^2 = (3.24 + 5.76) \text{ m}^2$$

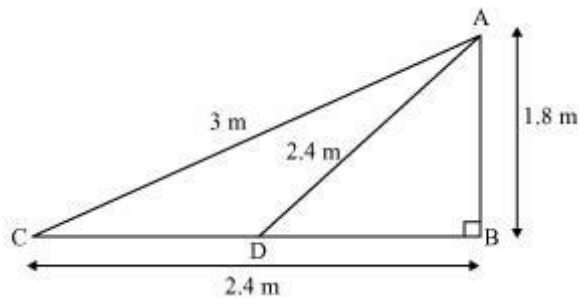
$$AB^2 = 9.00 \text{ m}^2$$

$$\Rightarrow AB = \sqrt{9} \text{ m} = 3 \text{ m}$$

Thus, the length of the string out is 3 m.

She pulls the string at the rate of 5 cm per second.

Therefore, string pulled in 12 seconds = $12 \times 5 = 60 \text{ cm} = 0.6 \text{ m}$



Let the fly be at point D after 12 seconds.

Length of string out after 12 seconds is AD.

$AD = AC - \text{String pulled by Nazima in 12 seconds}$

$$= (3.00 - 0.6) \text{ m}$$

$$= 2.4 \text{ m}$$

In $\triangle ADB$,

$$AB^2 + BD^2 = AD^2$$

$$(1.8 \text{ m})^2 + BD^2 = (2.4 \text{ m})^2$$

$$BD^2 = (5.76 - 3.24) \text{ m}^2 = 2.52 \text{ m}^2$$

$$BD = 1.587 \text{ m}$$

Horizontal distance of fly = $BD + 1.2 \text{ m}$

$$= (1.587 + 1.2) \text{ m}$$

$$= 2.787 \text{ m}$$

$$= 2.79 \text{ m}$$